21. Решить уравнение

$$(3x+4)\sqrt{-3x-2x^2-1}=0 \Leftrightarrow \left[\begin{array}{l}\begin{cases}3x+4=0\\ -3x-2x^2-1\ge 0\end{cases}\\ -3x-2x^2-1=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}x=-\dfrac{4}{3}\\ 2x^2+3x+1\le 0\end{cases}\\ 2x^2+3x+1=0\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l}\begin{cases}x=-\dfrac{4}{3}-\text{не подходит}\\ 2x^2+3x+1\le 0\end{cases}\\ 2x^2+3x+1=0\end{array}\right. \Leftrightarrow 2x^2+3x+1=0 \Leftrightarrow \left[\begin{array}{l}x=-1\\ x=-\dfrac{1}{2}\end{array}\right..$$

Ответ: $\left\{-1; -\dfrac{1}{2}\right\}$.

22. Решить уравнение

$$(4x-x^2-3)\sqrt{5x-8}=0 \Leftrightarrow \left[\begin{array}{l}\begin{cases}4x-x^2-3=0\\ 5x-8\ge 0\end{cases}\\ 5x-8=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}x^2-4x+3=0\\ x\ge\dfrac{8}{5}\end{cases}\\ x=\dfrac{8}{5}\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l}\begin{cases}\left[\begin{array}{l}x=1\\ x=3\end{array}\right.\\ x\ge\dfrac{8}{5}\end{cases}\\ x=\dfrac{8}{5}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=3\\ x=\dfrac{8}{5}\end{array}\right..$$

Ответ: $\left\{\dfrac{8}{5}; 3\right\}$.

23. Решить уравнение

$$1+\sin 3x=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2 \Leftrightarrow 1+\sin 3x=\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}+\sin^2\frac{x}{2} \Leftrightarrow$$

$$\Leftrightarrow \sin 3x+\sin x=0 \Leftrightarrow 2\sin 2x\cos x=0 \Leftrightarrow \sin 2x\cos x=0 \Leftrightarrow \left[\begin{array}{l}\sin 2x=0\\ \cos x=0\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l}2x=\pi k\\ x=\dfrac{\pi}{2}+\pi k : k\in\mathbb{Z}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi k}{2}\\ x=\dfrac{\pi}{2}+\pi k : k\in\mathbb{Z}\end{array}\right. \Leftrightarrow x=\frac{\pi k}{2} : k\in\mathbb{Z}.$$

Ответ: $\left\{\dfrac{\pi k}{2} : k\in\mathbb{Z}\right\}$.

24. Решить уравнение

$$2\sin^2 2x=(\cos x+\sin x)^2 \Leftrightarrow 2\sin^2 2x=\cos^2 x+2\sin x\cos x+\sin^2 x \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 2x-\sin 2x-1=0 \Leftrightarrow \left[\begin{array}{l}\sin 2x=1\\ \sin 2x=-\dfrac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}2x=\dfrac{\pi}{2}+2\pi k\\ 2x=-\dfrac{\pi}{6}+2\pi k\\ 2x=-\dfrac{5\pi}{6}+2\pi k : k\in\mathbb{Z}\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+\pi k \\ x=-\frac{\pi}{12}+\pi k \\ x=-\frac{5\pi}{12}+\pi k : k\in\mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{-\frac{\pi}{12}+\pi k; -\frac{5\pi}{12}+\pi k; \frac{\pi}{4}+\pi k : k\in\mathbb{Z}\right\}$.

25. Решить уравнение

$$\cos 9x-\cos 7x+\cos 3x-\cos x=0 \Leftrightarrow (\cos 9x-\cos x)-(\cos 7x-\cos 3x)=0 \Leftrightarrow$$

$$\Leftrightarrow -2\sin 5x\sin 4x+2\sin 5x\sin 2x=0 \Leftrightarrow \sin 5x(\sin 4x-\sin 2x)=0 \Leftrightarrow$$

$$\Leftrightarrow \sin 5x(2\sin x\cos 3x)=0 \Leftrightarrow \sin x\sin 5x\cos 3x=0 \Leftrightarrow \left[\begin{array}{l} \sin x=0 \\ \sin 5x=0 \\ \cos 3x=0 \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x=\pi k \\ 5x=\pi k \\ 3x=\frac{\pi}{2}+\pi k : k\in\mathbb{Z} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\pi k \\ x=\frac{\pi k}{5} \\ x=\frac{\pi}{6}+\frac{\pi k}{3} : k\in\mathbb{Z} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi k}{5} \\ x=\frac{\pi}{6}+\frac{\pi k}{3} : k\in\mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{\frac{\pi}{6}+\frac{\pi k}{3}; \frac{\pi k}{5} : k\in\mathbb{Z}\right\}$.

26. Решить уравнение

$$\cos 7x+\sin 8x=\cos 3x-\sin 2x \Leftrightarrow (\cos 7x-\cos 3x)+(\sin 8x+\sin 2x)=0 \Leftrightarrow$$

$$\Leftrightarrow -2\sin 5x\sin 2x+2\sin 5x\cos 3x=0 \Leftrightarrow \sin 5x(\sin 2x-\cos 3x)=0 \Leftrightarrow$$

$$\Leftrightarrow \sin 5x\left(\sin 2x-\sin\left(\frac{\pi}{2}-3x\right)\right)=0 \Leftrightarrow \sin 5x\left(2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=0 \Leftrightarrow$$

$$\Leftrightarrow \sin 5x\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)=0 \Leftrightarrow \left[\begin{array}{l} \sin 5x=0 \\ \sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)=0 \\ \cos\left(\frac{\pi}{4}-\frac{x}{2}\right)=0 \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} 5x=\pi k \\ \frac{5x}{2}-\frac{\pi}{4}=\pi k \\ \frac{\pi}{4}-\frac{x}{2}=\frac{\pi}{2}+\pi k : k\in\mathbb{Z} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi k}{5} \\ \frac{5x}{2}=\frac{\pi}{4}+\pi k \\ \frac{x}{2}=-\frac{\pi}{4}-\pi k : k\in\mathbb{Z} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi k}{5} \\ x=\frac{\pi}{10}+\frac{2\pi k}{5} \\ x=-\frac{\pi}{2}-2\pi k : k\in\mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{\frac{\pi k}{5}; \frac{\pi}{10}+\frac{2\pi k}{5}; -\frac{\pi}{2}-2\pi k : k\in\mathbb{Z}\right\}$.

27. Решить уравнение

$$\sin x-\sin 2x+\sin 5x+\sin 8x \Leftrightarrow (\sin x+\sin 5x)+(\sin 8x-\sin 2x)=0 \Leftrightarrow$$

$$\Leftrightarrow 2\sin 3x\cos 2x+2\sin 3x\cos 5x=0 \Leftrightarrow \sin 3x(\cos 2x+\cos 5x)=0 \Leftrightarrow$$

$$\Leftrightarrow \sin 3x\left(2\cos\frac{7x}{2}\cos\frac{3x}{2}\right)=0 \Leftrightarrow \sin 3x\cos\frac{7x}{2}\cos\frac{3x}{2}=0 \Leftrightarrow \left[\begin{array}{l}\sin 3x=0\\ \cos\frac{7x}{2}=0\\ \cos\frac{3x}{2}=0\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l}3x=\pi k\\ \frac{7x}{2}=\frac{\pi}{2}+\pi k\\ \frac{3x}{2}=\frac{\pi}{2}+\pi k : k\in\mathbb{Z}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi k}{3}\\ x=\frac{\pi}{7}+\frac{2\pi k}{7}\\ x=\frac{\pi}{3}+\frac{2\pi k}{3} : k\in\mathbb{Z}\end{array}\right. .$$

Ответ: $\left\{\frac{\pi k}{3}; \frac{\pi}{7}+\frac{2\pi k}{7}; \frac{\pi}{3}+\frac{2\pi k}{3} : k\in\mathbb{Z}\right\}$.

28. Решить уравнение

$$\sin x+\sin 3x-\sin 5x-\sin 7x \Leftrightarrow (\sin x+\sin 3x)-(\sin 5x+\sin 7x)=0 \Leftrightarrow$$

$$\Leftrightarrow 2\sin 2x\cos x-2\sin 6x\cos x=0 \Leftrightarrow \cos x(\sin 2x-\sin 6x)=0 \Leftrightarrow$$

$$\Leftrightarrow \cos x(-2\sin 2x\cos 4x)=0 \Leftrightarrow \cos x\sin 2x\cos 4x=0 \Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \sin 2x=0\\ \cos 4x=0\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+\pi k\\ 2x=\pi k\\ 4x=\frac{\pi}{2}+\pi k : k\in\mathbb{Z}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+\pi k\\ x=\frac{\pi k}{2}\\ x=\frac{\pi}{8}+\frac{\pi k}{4} : k\in\mathbb{Z}\end{array}\right. .$$

Ответ: $\left\{\frac{\pi}{2}+\pi k; \frac{\pi k}{2}; \frac{\pi}{8}+\frac{\pi k}{4} : k\in\mathbb{Z}\right\}$.

29. Решить уравнение

$$\cos 2x+\cos 6x+2\sin^2 x=1 \Leftrightarrow \cos 2x+\cos 6x=1-2\sin^2 x \Leftrightarrow$$

$$\Leftrightarrow \cos 2x+\cos 6x=\cos 2x \Leftrightarrow \cos 6x=0 \Leftrightarrow 6x=\frac{\pi}{2}+\pi k : k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=\frac{\pi}{12}+\frac{\pi k}{6} : k\in\mathbb{Z}.$$

Ответ: $\left\{\frac{\pi}{12}+\frac{\pi k}{6} : k\in\mathbb{Z}\right\}$.

30. Решить уравнение

$$4\cos x\sin x+(\operatorname{tg} x+\operatorname{ctg} x)=0 \Leftrightarrow 2(2\sin x\cos x)+\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=0 \Leftrightarrow$$

$$\Leftrightarrow 2\sin 2x+\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=0 \Leftrightarrow 2\sin 2x+\frac{1}{\sin x\cos x}=0 \Leftrightarrow 2\sin 2x+\frac{2}{\sin 2x}=0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}\sin 2x\neq 0\\ 2\sin^2 2x+2=0\end{cases} \text{— решений нет.}$$

Ответ: решений нет.