161. Решить уравнение с модулем

$$|\sin x| = \sin x + 2\cos x \Leftrightarrow \left[\begin{array}{l}\begin{cases}\sin x \ge 0\\ \sin x = \sin x + 2\cos x\end{cases}\\ \begin{cases}\sin x \le 0\\ -\sin x = \sin x + 2\cos x\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}\sin x \ge 0\\ \cos x = 0\end{cases}\\ \begin{cases}\sin x \le 0\\ \sin x + \cos x = 0\end{cases}\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l}\begin{cases}\sin x \ge 0\\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z}\end{cases}\\ \begin{cases}\sin x \le 0\\ \operatorname{tg} x + 1 = 0\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + 2\pi k\\ \begin{cases}\sin x \le 0\\ x = -\dfrac{\pi}{4} + \pi k : k \in \mathbb{Z}\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + 2\pi k\\ x = -\dfrac{\pi}{4} + 2\pi k : k \in \mathbb{Z}\end{array}\right..$$

Ответ: $\left\{-\dfrac{\pi}{4} + 2\pi k;\ \dfrac{\pi}{2} + 2\pi k : k \in \mathbb{Z}\right\}$.

162. Решить уравнение с модулем

$$|\operatorname{tg} x| = \operatorname{tg} x - \frac{1}{\cos x} \Leftrightarrow \left[\begin{array}{l}\begin{cases}\operatorname{tg} x \ge 0\\ \operatorname{tg} x = \operatorname{tg} x - \dfrac{1}{\cos x}\end{cases}\\ \begin{cases}\operatorname{tg} x < 0\\ -\operatorname{tg} x = \operatorname{tg} x - \dfrac{1}{\cos x}\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}\operatorname{tg} x \ge 0\\ \dfrac{1}{\cos x} = 0\end{cases} - \text{решений нет}\\ \begin{cases}\operatorname{tg} x < 0\\ 2\sin x - 1 = 0\end{cases}\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \begin{cases}\operatorname{tg} x < 0\\ \sin x = \dfrac{1}{2}\end{cases} \Leftrightarrow \begin{cases}\operatorname{tg} x < 0\\ \left[\begin{array}{l} x = \dfrac{\pi}{6} + 2\pi k\\ x = \dfrac{5\pi}{6} + 2\pi k : k \in \mathbb{Z}\end{array}\right.\end{cases} \Leftrightarrow x = \frac{5\pi}{6} + 2\pi k : k \in \mathbb{Z}.$$

Ответ: $\left\{\dfrac{5\pi}{6} + 2\pi k : k \in \mathbb{Z}\right\}$.

163. Решить уравнение с модулем

$$|\cos x| = \cos x - 2\sin x \Leftrightarrow \left[\begin{array}{l}\begin{cases}\cos x \ge 0\\ \cos x = \cos x - 2\sin x\end{cases}\\ \begin{cases}\cos x < 0\\ -\cos x = \cos x - 2\sin x\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}\cos x \ge 0\\ \sin x = 0\end{cases}\\ \begin{cases}\cos x < 0\\ \sin x - \cos x = 0\end{cases}\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x = 2\pi k\\ \begin{cases}\cos x < 0\\ \operatorname{tg} x - 1 = 0\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 2\pi k : k \in \mathbb{Z}\\ \begin{cases}\cos x < 0\\ x = \dfrac{\pi}{4} + \pi k : k \in \mathbb{Z}\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 2\pi k\\ x = -\dfrac{3\pi}{4} + 2\pi k : k \in \mathbb{Z}\end{array}\right..$$

Ответ: $\left\{2\pi k;\ -\dfrac{3\pi}{4} + 2\pi k : k \in \mathbb{Z}\right\}$.

164. Решить уравнение с модулем

$$|\operatorname{ctg} x| = \operatorname{ctg} x + \frac{1}{\sin x} \Leftrightarrow \left[\begin{array}{l}\begin{cases}\operatorname{ctg} x \ge 0\\ \operatorname{ctg} x = \operatorname{ctg} x + \dfrac{1}{\sin x}\end{cases}\\ \begin{cases}\operatorname{ctg} x < 0\\ -\operatorname{ctg} x = \operatorname{ctg} x + \dfrac{1}{\sin x}\end{cases}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\begin{cases}\operatorname{ctg} x \ge 0\\ \dfrac{1}{\sin x} = 0\end{cases} - \text{решений нет}\\ \begin{cases}\operatorname{ctg} x < 0\\ 2\cos x + 1 = 0\end{cases}\end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \operatorname{ctg} x < 0 \\ \cos x = -\dfrac{1}{2} \end{cases} \Leftrightarrow \begin{cases} \operatorname{ctg} x < 0 \\ \left[\begin{array}{l} x = \dfrac{2\pi}{3} + 2\pi k \\ x = \dfrac{4\pi}{3} + 2\pi k : k \in \mathbb{Z} \end{array} \right. \end{cases} \Leftrightarrow x = \frac{2\pi}{3} + 2\pi k : k \in \mathbb{Z}.$$

Ответ: $\left\{ \dfrac{2\pi}{3} + 2\pi k : k \in \mathbb{Z} \right\}$.

165. Решить уравнение с модулем

$$\cos x = |\cos x| \left(x + \frac{3}{2} \right)^2 \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \cos x = \cos x \left(x + \dfrac{3}{2} \right)^2 \end{cases} \\ \begin{cases} \cos x = 0 \\ 0 = 0 \end{cases} \\ \begin{cases} \cos x < 0 \\ -\cos x = \cos x \left(x + \dfrac{3}{2} \right)^2 \end{cases} \end{array} \right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \left(x + \dfrac{3}{2} \right)^2 = 1 \end{cases} \\ \begin{cases} \cos x = 0 \\ 0 = 0 \end{cases} - \text{верно} \\ \begin{cases} \cos x < 0 \\ \left(x + \dfrac{3}{2} \right)^2 = -1 \end{cases} - \text{решений нет} \end{array} \right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \left[\begin{array}{l} x + \dfrac{3}{2} = 1 \\ x + \dfrac{3}{2} = -1 \end{array} \right. \end{cases} \\ \cos x = 0 \end{array} \right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \left[\begin{array}{l} x = -\dfrac{1}{2} \\ x = -\dfrac{5}{2} \end{array} \right. \end{cases} \\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \end{array} \right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos\left(-\dfrac{1}{2} \right) > 0 - \text{верно} \\ x = -\dfrac{1}{2} \end{cases} \\ \begin{cases} \cos\left(-\dfrac{5}{2} \right) > 0 - \text{неверно} \\ x = -\dfrac{5}{2} \end{cases} \\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \end{array} \right. \Leftrightarrow \left[\begin{array}{l} x = -\dfrac{1}{2} \\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \end{array} \right..$$

Ответ: $\left\{ -\dfrac{1}{2}; \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \right\}$.

166. Решить уравнение с модулем

$$|\cos x| = \cos x (x-2)^2 \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \cos x = \cos x (x-2)^2 \end{cases} \\ \begin{cases} \cos x = 0 \\ 0 = 0 \end{cases} \\ \begin{cases} \cos x < 0 \\ -\cos x = \cos x (x-2)^2 \end{cases} \end{array} \right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ (x-2)^2 = 1 \end{cases} \\ \begin{cases} \cos x = 0 \\ 0 = 0 - \text{верно} \end{cases} \\ \begin{cases} \cos x < 0 \\ (x-2)^2 = -1 - \text{решений нет} \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \left[\begin{array}{l} x-2 = 1 \\ x-2 = -1 \end{array}\right. \end{cases} \\ \cos x = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \left[\begin{array}{l} x = 3 \\ x = 1 \end{array}\right. \end{cases} \\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos(1) > 0 - \text{верно} \\ x = 1 \end{cases} \\ \begin{cases} \cos(3) > 0 - \text{неверно} \\ x = 3 \end{cases} \\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 1 \\ x = \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{1; \dfrac{\pi}{2} + \pi k : k \in \mathbb{Z}\right\}$.

167. Решить уравнение с модулем

$$\cos x = |\sin x| \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x \ge 0 \\ \cos x - \sin x = 0 \end{cases} \\ \begin{cases} \sin x \le 0 \\ \cos x + \sin x = 0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x \ge 0 \\ \operatorname{tg} x = 1 \end{cases} \\ \begin{cases} \sin x \le 0 \\ \operatorname{tg} x + 1 = 0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x \ge 0 \\ x = \dfrac{\pi}{4} + \pi k \end{cases} \\ \begin{cases} \sin x \le 0 \\ x = -\dfrac{\pi}{4} + \pi k : k \in \mathbb{Z} \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{4} + 2\pi k \\ x = -\dfrac{\pi}{4} + 2\pi k : k \in \mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{-\dfrac{\pi}{4} + 2\pi k; \dfrac{\pi}{4} + 2\pi k : k \in \mathbb{Z}\right\}$.

168. Решить уравнение с модулем

$$\sqrt{3}\sin x = |\cos x| \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x = 0 \\ \sin x = 0 \end{cases} - \text{невозможно} \\ \begin{cases} \cos x > 0 \\ \sqrt{3}\sin x = \cos x \end{cases} \\ \begin{cases} \cos x < 0 \\ \sqrt{3}\sin x = -\cos x \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \sqrt{3}\operatorname{tg} x = 1 \end{cases} \\ \begin{cases} \cos x < 0 \\ \sqrt{3}\operatorname{tg} x = -1 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ \operatorname{tg} x = \dfrac{1}{\sqrt{3}} \end{cases} \\ \begin{cases} \cos x < 0 \\ \operatorname{tg} x = -\dfrac{1}{\sqrt{3}} \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \cos x > 0 \\ x = \dfrac{\pi}{6} + \pi k \end{cases} \\ \begin{cases} \cos x < 0 \\ x = -\dfrac{\pi}{6} + \pi k : k \in \mathbb{Z} \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{6} + 2\pi k \\ x = \dfrac{5\pi}{6} + 2\pi k : k \in \mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{x = \dfrac{\pi}{6} + 2\pi k; \dfrac{5\pi}{6} + 2\pi k : k \in \mathbb{Z}\right\}$.

169. Решить уравнение с модулем

$$2\sin^2 x = |\sin x| \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x \ge 0 \\ 2\sin^2 x - \sin x = 0 \end{cases} \\ \begin{cases} \sin x \le 0 \\ 2\sin^2 x + \sin x = 0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x \ge 0 \\ \sin x(2\sin x - 1) = 0 \end{cases} \\ \begin{cases} \sin x \le 0 \\ \sin x(2\sin x + 1) = 0 \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x \ge 0 \\ \left[\begin{array}{l} x = \pi k \\ x = \dfrac{\pi}{6} + 2\pi k \\ x = \dfrac{5\pi}{6} + 2\pi k \end{array}\right. \end{cases} \\ \begin{cases} \sin x \le 0 \\ \left[\begin{array}{l} x = \pi k \\ x = -\dfrac{\pi}{6} + 2\pi k \\ x = -\dfrac{5\pi}{6} + 2\pi k : k \in \mathbb{Z} \end{array}\right. \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \pi k \\ x = \dfrac{\pi}{6} + \pi k \\ x = -\dfrac{\pi}{6} + \pi k : k \in \mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{\pi k; \dfrac{\pi}{6} + \pi k; -\dfrac{\pi}{6} + \pi k : k \in \mathbb{Z}\right\}$.

170. Решить уравнение с модулем

$$2\cos^2 x = |\sin x| \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x = 0 \\ 2\cos^2 x = 0 \end{cases} - \text{невозможно} \\ \begin{cases} \sin x > 0 \\ 2\cos^2 x - \sin x = 0 \end{cases} \\ \begin{cases} \sin x < 0 \\ 2\cos^2 x + \sin x = 0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x > 0 \\ 2\sin^2 x + \sin x - 2 = 0 \end{cases} \\ \begin{cases} \sin x < 0 \\ 2\sin^2 x - \sin x - 2 = 0 \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x > 0 \\ \left[\begin{array}{l} \sin x = \dfrac{-1-\sqrt{17}}{4} - \text{решений нет} \\ \sin x = \dfrac{-1+\sqrt{17}}{4} \end{array}\right. \end{cases} \\ \begin{cases} \sin x < 0 \\ \left[\begin{array}{l} \sin x = \dfrac{1-\sqrt{17}}{4} \\ \sin x = \dfrac{1+\sqrt{17}}{4} - \text{решений нет} \end{array}\right. \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \sin x > 0 \\ x = (-1)^k \arcsin\left(\dfrac{-1+\sqrt{17}}{4}\right) + 2\pi k \end{cases} \\ \begin{cases} \sin x < 0 \\ x = (-1)^k \arcsin\left(\dfrac{1-\sqrt{17}}{4}\right) + 2\pi k : k \in \mathbb{Z} \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x = (-1)^k \arcsin\left(\dfrac{-1+\sqrt{17}}{4}\right) + \pi k \\ x = (-1)^k \arcsin\left(\dfrac{1-\sqrt{17}}{4}\right) + \pi k : k \in \mathbb{Z} \end{array}\right. .$$

Ответ: $\left\{(-1)^k \arcsin\left(\dfrac{-1+\sqrt{17}}{4}\right) + \pi k; (-1)^k \arcsin\left(\dfrac{1-\sqrt{17}}{4}\right) + \pi k : k \in \mathbb{Z}\right\}$.